Math 7760 – Homework 6 – Due: October 21, 2022

Practice Problems:

Problem 1. If you are interested in learning about the connection between greedy algorithms and matroids, read Section 1.8 in Oxley.

Problem 2. Use matroid duality to prove the hard direction of the hyperplane axioms.

Problem 3. What is the dual of $U_{r,n}$?

Problems to write up:

Problem 4. Let M be a matroid with circuit C and cocircuit C^*. Prove that $|C \cap C^*| \neq 1$.

Problem 5. Given a matroid M on ground set E and a basis B, prove that for each $e \in E \setminus B$, there exists a unique circuit in $B \cup \{e\}$. Use this to prove that for any connected graph G with spanning tree T, for each $e \in T$ (this is not a typo), there exists a unique minimal cut of G whose only edge in common with T is e.

Problem 6. Let $A \in \mathbb{F}^{r \times n}$ have rank r and let $B \in \mathbb{F}^{(n-r) \times n}$ have rank $n - r$ and assume that $AB^T = 0$. Show that there exists a nonzero $\lambda \in \mathbb{F}$ such that for any $S \subseteq E$ of size r, if A_S denotes the column-submatrix of A on columns indexed by S and $B_{\{1,\ldots,n\} \setminus S}$ denotes the column-submatrix of B on columns indexed by $\{1, \ldots, n\} \setminus S$, then $\det(A_S) = \lambda \det(B_{\{1,\ldots,n\} \setminus S})$.